Worksheet

Linear equations 4

worked example 1

Solve the linear equation $\frac{3x+7}{2}=\frac{5-2x}{3}$.

Steps	Solution
1. Cross multiply by both denominators.	$3(3x+7)=2(5-2x)$
2. Expand brackets.	$9x+21=10-4x$
3. Add $4x$ to both sides.	$13x+21=10$
4. Subtract 21 from both sides.	$13x=-11$
5. Divide both sides by 13.	$\therefore x=-\frac{11}{13}$

worked example 2

Solve the linear equation $\frac{12}{6-3x}=-2$.

Steps	Solution
1. Multiply both sides by the denominator.	$12=-2(6-3x)$
2. Expand brackets.	$12=-12+6x$
3. Add 12 to both sides.	$24=6x$
4. Divide both sides by 6.	$\therefore x=4$

exercise

1 Solve the following linear equations.

(a) $\frac{2x+7}{5}=9$ (b) $\frac{-2(7x-1)}{5}=3$ (c) $\frac{5(2x-3)}{3}+\frac{5}{6}=0$

(d) $\frac{12}{2x+3}=-5$ (e) $\frac{3}{2x-7}=4$ (f) $\frac{5x+1}{4}=\frac{3-2x}{7}$

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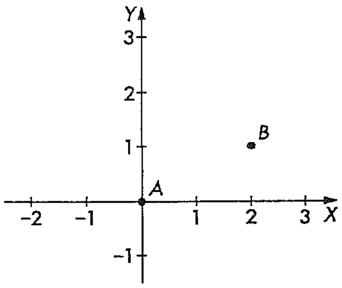
Worksheet

Linear graphs

Cartesian coordinates

Given the following diagram we can identify the location of A and B:

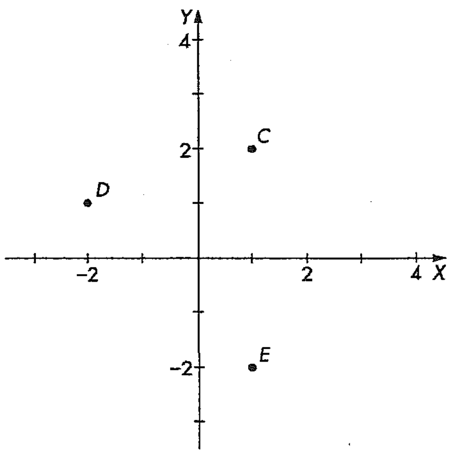


By convention, we must start at the intersection of the two axes. This point is also called the origin. In the grid, A is at the origin and its coordinates are (0, 0). To describe the location of point B, remember we always move *right* first and then *up*. So, B is 2 units to the right, and 1 unit up. It can be written as (2, 1). A set of coordinates locating a point is always presented in brackets.

Note: The movement to the *right* corresponds to the x value. The movement *up* corresponds to the y value. Coordinates are always shown (x, y) with the right/left movement first and the up/down movement second.

exercise

1



Complete the following statements about the points C, D and E.

(a) C is 1 unit right and ...... units up. The coordinates of C are (1, ...).

(b) D is 2 units .... and ...... unit .... The coordinates of D are (-2, ...).

(c) E is ...... unit .... and ...... units .... The coordinates of E are (......, -2).

2 Describe how you would plot each of the following points, starting from the origin.

(a) Point F (4, 1)

4 units right, ... unit up.

(b) Point G (-2, 4)

...... units to the ...., ... units ....

(c) Point H (3, -2)

...... units to the ...., ... units ....

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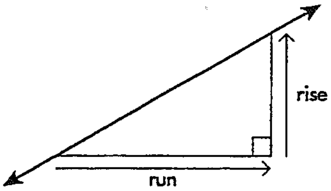
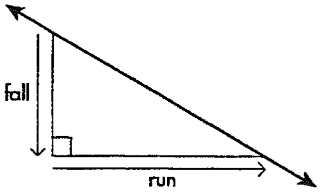
Gradient

The gradient, m, of a line is given by:

$$m = \frac{\text{rise}}{\text{run}}$$



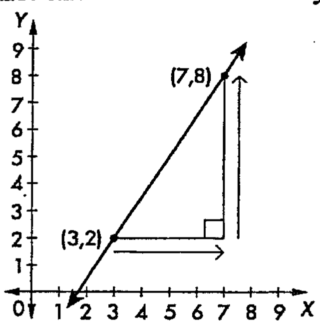


Note: fall = -rise

exercise

3 In each of the following examples, find:
(i) the rise (or fall) (ii) the run (iii) the gradient (m).
(The first one has been done for you.)

(a)

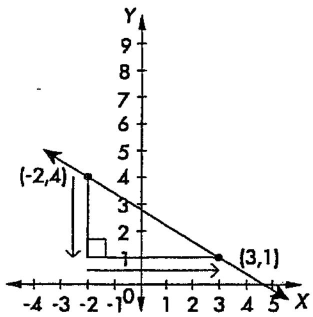


(i) rise = 8 – 2 = 6

(ii) run = 7 – 3 = 4

(iii) $m = \frac{6}{4} = \frac{3}{2}$

(b)

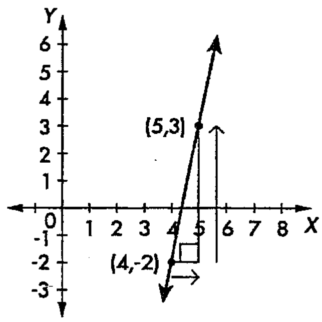


(i) rise = 1 – 4 = ...

(ii) run = 3 – -2 = ...

(iii) $m = \frac{-3}{5}$

(c)

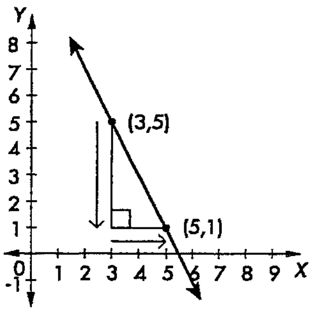


(i) rise = ... – -2 = ...

(ii) run = ... – ..... = 1

(iii) $m = \frac{\cdots\cdots}{\cdots\cdots} = \ldots\ldots$

(d)

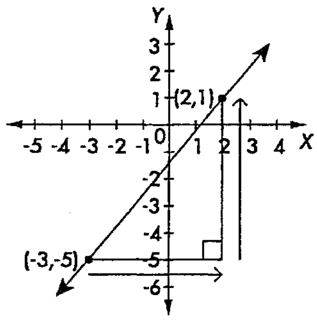


(i) rise = ... – 5 = ...

(ii) run = ... – ..... = ...

(iii) $m = \frac{-4}{\ldots\ldots} = \ldots\ldots$

(e)

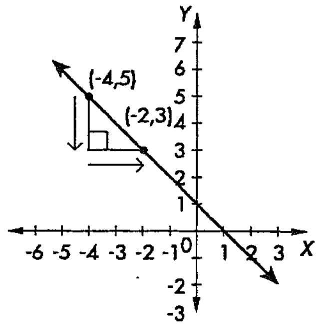


(i) rise = ... – ..... = ...

(ii) run = ... – ..... = ...

(iii) $m = \frac{\cdots\cdots}{\cdots\cdots}$

(f)



(i) rise = ... – ..... = ...

(ii) run = ... – -4 = ...

(iii) $m = \frac{\cdots\cdots}{2} = \ldots\ldots$

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Finding the equation of a straight line

Remember: $y = mx + c$ and $y - y_1 = m(x - x_1)$.

worked example 1

Find the equation of the straight line with gradient m = -2 that passes through the point (2, 6).

Steps	**Solution**
1. Determine x_1 and y_1 values and substitute into the formula.	$x_1 = 2$, $y_1 = 6$ $y - 6 = -2(x - 2)$
2. Expand the brackets.	$y - 6 = -2x + 4$
3. Add 6 to both sides and write the equation of the straight line.	$y = -2x + 10$

worked example 2

Find the equation of the straight line with gradient $m = \frac{1}{2}$ that passes through the point $\left(\frac{-3}{4}, \frac{2}{3}\right)$.

Steps	**Solution**
1. Determine x_1 and y_1 values and substitute into the formula.	$x_1 = \frac{-3}{4}$, $y_1 = \frac{2}{3}$ $y - \frac{2}{3} = \frac{1}{2}\left(x + \frac{3}{4}\right)$
2. Expand the brackets.	$y - \frac{2}{3} = \frac{1}{2}x + \frac{3}{8}$
3. Add $\frac{2}{3}$ to both sides and write the equation of the straight line.	$y = \frac{1}{2}x + \frac{25}{24}$

exercise

4 Find the equation of the straight lines that pass through the following points and have the given gradients.

(a) (1, 5), $m = 3$

(b) (2, 3), $m = -2$

(c) (-1, 2), $m = \frac{1}{2}$

(d) (3, -1), $m = \frac{3}{4}$

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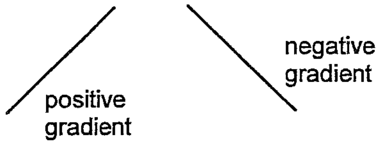
Worksheet

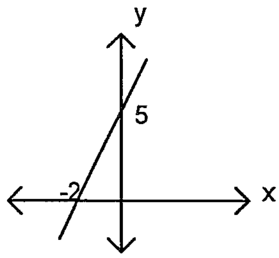
Gradient of linear equations

Remember: gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$



example 1

Calculate the gradient of the following line.



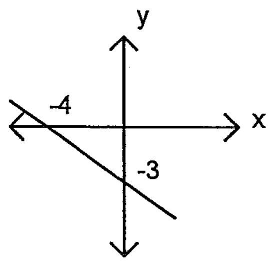
Solution

Slope is positive
$(x_1, y_1) = (-2, 0)$
$(x_2, y_2) = (0, 5)$

$$\text{gradient, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{0 - -2} = \frac{5}{2}$$

example 2

Calculate the gradient of the following line.



Solution

Slope is negative
$(x_1, y_1) = (-4, 0)$
$(x_2, y_2) = (0, -3)$

$$\text{gradient, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{0 - -4} = \frac{-3}{4}$$

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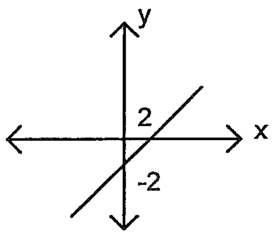
exercise

1 Calculate the gradients of the following lines.

(a)



$(x_1, y_1) = (0, -2)$

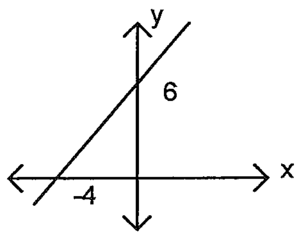
$(x_2, y_2) = (\ldots\ldots, \ldots\ldots)$

$$\text{gradient, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\ldots\ldots - -2}{\ldots\ldots - 0}$$

$$= \ldots\ldots$$

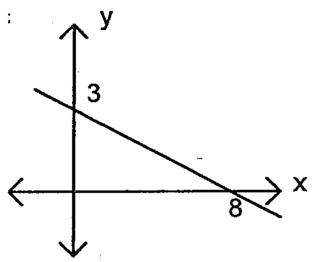
(b)



$(x_1, y_1) = (\ldots\ldots, \ldots\ldots)$

$(x_2, y_2) = (\ldots\ldots, \ldots\ldots)$

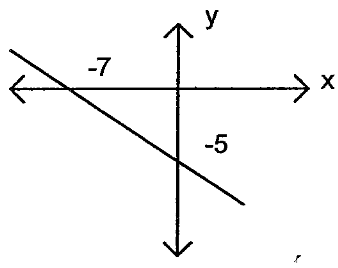
(c)



$(x_1, y_1) = (\ldots\ldots, \ldots\ldots)$

$(x_2, y_2) = (\ldots\ldots, \ldots\ldots)$

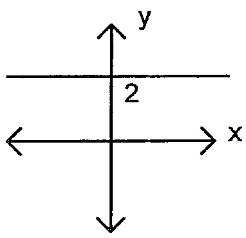
(d)



$(x_1, y_1) = (\ldots\ldots, \ldots\ldots)$

$(x_2, y_2) = (\ldots\ldots, \ldots\ldots)$

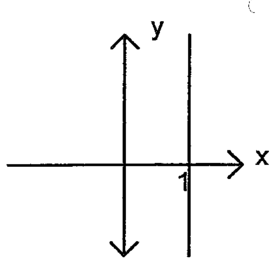
(e)



$(x_1, y_1) = (\ldots\ldots, \ldots\ldots)$

$(x_2, y_2) = (\ldots\ldots, \ldots\ldots)$

(f)



$(x_1, y_1) = (\ldots\ldots, \ldots\ldots)$

$(x_2, y_2) = (\ldots\ldots, \ldots\ldots)$

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Worksheet

Finding the y-intercept and the gradient from the equation

The equation

The equation of a straight line is $y = mx + c$, where m = gradient and c = the y-intercept. The y-intercept is the point where $x = 0$ (or where the line crosses the y-axis).

exercise

1 Find the gradient (m) and the y-intercept (c) for each of the following examples. (The first one has been done for you.)

(a) $y = 2x + 3$

$\uparrow \quad \uparrow$

$m = 2 \quad c = 3$

(b) $y = 3x - 5$

$\uparrow \quad \uparrow$

$m = \ldots \quad c = \ldots$

(c) $y = 5x + 1$

$\uparrow \quad \uparrow$

$m = \ldots \quad c = \ldots$

(d) $y = 2x + 5$

$m = \ldots\ldots, c = \ldots\ldots$

(e) $y = -x + 1$

$m = \ldots\ldots, c = \ldots\ldots$

(f) $y = 7x - 11$

$m = \ldots\ldots, c = \ldots\ldots$

(g) $y = 3 - 2x$

$= -2x + 3$

[reordered]

$m = \ldots\ldots, c = \ldots\ldots$

(h) $y = 7 - 5x$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

[reordered]

$m = \ldots\ldots, c = \ldots\ldots$

(i) $y = -5 - 3x$

$= \ldots\ldots\ldots\ldots\ldots\ldots$

[reordered]

$m = \ldots\ldots, c = \ldots\ldots$

Reordering the equation

At times, the equation is not written in the form $y = mx + c$.

e.g. $2x + 3y = 6$ In this case, the equation must be rearranged so that it is in the $y = mx + c$ form.

$2x + 3y = 6$

$3y = 6 - 2x$ [by subtracting $2x$ from both sides]

$y = \frac{6}{3} - \frac{2}{3}x$ [by dividing both sides by 3]

$= \frac{-2}{3}x + \frac{6}{3}$ [by reordering]

$= \frac{-2}{3}x + 2$ [*Note:* $\frac{6}{3} = 2$]

So: $m = \frac{-2}{3}$ and $c = 2$

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exercise

2 Rearrange each of the following equations, so that you can find the gradient (m) and the y-intercept (c).

(a) $3x + 4y = 7$

$4y = 7 - \ldots\ldots$ [by subtracting $3x$ from both sides]

$y = \frac{7}{4} - \frac{3x}{4}$ [by dividing both sides by 4]

$= \frac{\ldots\ldots}{\ldots\ldots} + \frac{7}{4}$ [by reordering]

$\therefore$ $m = \ldots\ldots, c = \ldots\ldots$

(b) $7x + 2y = 8$

$2y = \ldots\ldots - \ldots\ldots$ [by subtracting ...... from both sides]

$y = \frac{\ldots\ldots}{\ldots\ldots} - \frac{\ldots\ldots}{\ldots\ldots}$ [by dividing both sides by ......]

$= \frac{-7}{2}x + \ldots\ldots$ [by reordering and simplifying]

$\therefore$ $m = \ldots\ldots, c = \ldots\ldots$

(c) $2x - 3y = 6$

$-3y = 6 - \ldots\ldots$ [by subtracting ...... from both sides]

$y = \frac{6}{-3} - \frac{2x}{-3}$ [by dividing both sides by ......]

$y = -2 + \frac{2}{3}x$ [by simplifying]

$y = \ldots\ldots - \ldots\ldots$ [by reordering]

$\therefore$ $m = \ldots\ldots, c = \ldots\ldots$

(d) $4x - 5y = 3$

$-5y = \ldots\ldots - \ldots\ldots$ [by subtracting ...... from both sides]

$y = \frac{\ldots\ldots}{\ldots\ldots} - \frac{\ldots\ldots}{\ldots\ldots}$ [by dividing both sides by ......]

$y = \ldots\ldots + \ldots\ldots$ [by reordering]

$\therefore$ $m = \ldots\ldots, c = \ldots\ldots$

(e) $3x + 2y = -12$

$2y = \ldots\ldots - \ldots\ldots$ [by subtracting ...... from both sides]

$y = \frac{\ldots\ldots}{\ldots\ldots} - \frac{\ldots\ldots}{\ldots\ldots}$ [by dividing both sides by ......]

$y = \ldots\ldots - \ldots\ldots$ [by reordering]

$\therefore$ $m = \ldots\ldots, c = \ldots\ldots$

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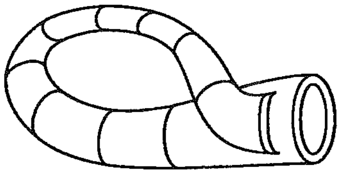
Worksheet

Gradient and intercepts

Felix Klein invented a special bottle, given the name of Klein bottle, in 1882. The bottle can be made by bending a tube into a loop and inserting one end into a hole in the side of the tube. The two open ends are then joined together. Forming a bottle in this fashion gives it some fascinating properties.



What are some of the properties of the Klein bottle?

Find out the answer by solving the following, showing all your working in the space provided. Write the question's code letter above its solution in the tables on the next page.

Find the gradient of the line joining each of the following pairs of points.

N (4, 0) and (0, 20)	**F** (1, 3) and (4, 15)
T (-3, 2) and (-1, 8)	**G** (5, 11) and (7, 3)
I (5, 2) and (4, 8)	**H** (3, -2) and (5, 14)
S (-2, 11) and (-3, 2)	**R** (1, 3) and (4, 3)

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For each of the following linear equations determine what is indicated in brackets.

U $3x + 4y = 24$ (y-intercept)	**D** $6x - 2y = 12$ (x-intercept)
O $x + y = 7$ (gradient)	**E** $x - 6y = 18$ (y-intercept)
W $7x - 3y = -21$ (y-intercept)	**C** $6x - 4y = -12$ (x-intercept)
A $5x - y = 5$ (x-intercept)	

What are some of the properties of the Klein bottle?

-6	3		-6	9		1		-1	-5	-3	–	9	-6	2	-3	2

9	6	0	4	1	-2	-3		7	-6	3	8		-5	-1

-3	2	-4	-3	9		-5	-1		-6	-5	9	-6	2	-3

1	-5	2		-5	-1		-1	6	3	9	-6	2	-3

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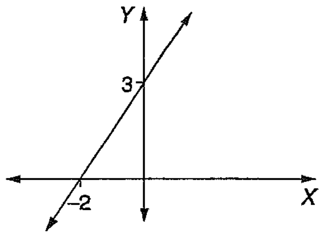
Worksheet

Finding the equation of a straight line 1

The equation of a straight line is easily obtained if the gradient and y-intercept are known. We simply substitute the given values of m and c into the equation $y = mx + c$.

worked example

Find the equation of the linear graph shown.



Steps	***Solution***
1. Identify the y-intercept, c.	$c = 3$
2. Calculate the gradient, $m = \frac{\text{rise}}{\text{run}}$.	The graph rises from (-2, 0) to (0, 3). $m = \frac{3-0}{0-(-2)} = \frac{3}{2}$
3. Substitute the value for the gradient into the equation $y = mx + c$.	$y = mx + c$ $y = \frac{3}{2}x + 3$

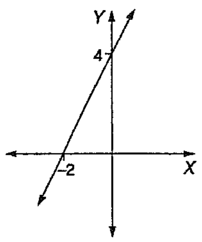
Lines parallel to the axes

- If $m = 0$, then the line is horizontal (i.e. parallel to the x-axis), and the equation of the straight line is $y = c$.
- If a line is vertical (i.e. parallel to the y-axis), its gradient is undefined and its rule is $x = d$, where d is the x-intercept.

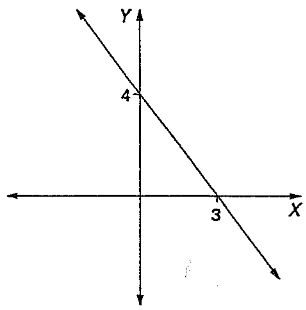
exercise

1 Find the equation of each of the following straight lines.

(a)



(b)



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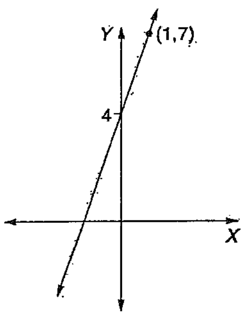
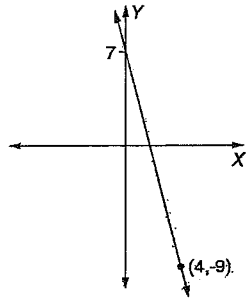
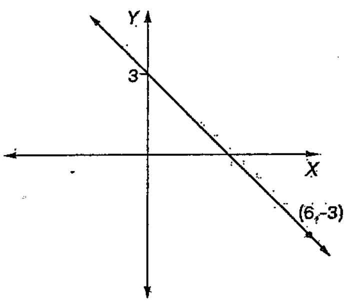
(c)



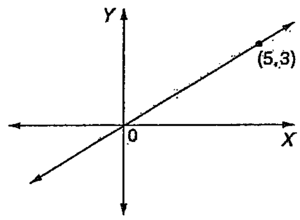
(d)



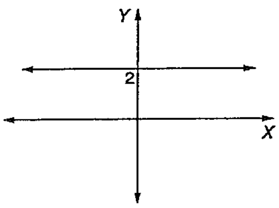
(e)



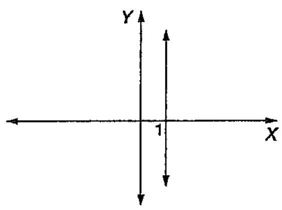
(f)



(g)



(h)



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Worksheet

Finding the equation of a straight line 2

Remember: $y = mx + c$.

worked example 1

Find the equation of the straight line with gradient $m = 2$ that passes through the point (1, 5).

Steps	Solution
1. Substitute the value for m into $y = mx + c$.	$y = 2x + c$
2. To find c, substitute the known point into the equation and solve for c.	$5 = 2 \times 1 + c$ $5 = 2 + c$ $c = 3$
3. Substitute m and c into $y = mx + c$.	$y = 2x + 3$

worked example 2

Find the equation of the straight line that passes through the points (2, 6) and (-1, 2).

Steps	Solution
1. Find the gradient, m, of the line.	$m = \frac{2-6}{-1-2}$ $= \frac{-4}{-3}$ $= \frac{4}{3}$
2. Substitute the value for m into $y = mx + c$.	$y = \frac{4}{3}x + c$
3. To find c, substitute one of the known points into the equation and solve for c.	$2 = \frac{4}{3} \times -1 + c$ $2 = -\frac{4}{3} + c$ $c = \frac{10}{3}$
4. Substitute m and c into $y = mx + c$.	$y = \frac{4}{3}x + \frac{10}{3}$

Note that the above equation could also be written as $3y = 4x + 10$ or $3y - 4x = 10$.

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exercise

1 Find the equation of each of the straight lines that have the given gradients and y-intercepts.

(a) $m = 2, c = -3$

$y = \ldots x + \ldots$

(b) $m = -1, c = 5$

(c) $m = 0, c = -3$

(d) $m = \frac{1}{2}, c = 6$

2 Find the equation of each of the straight lines that pass through the following points and have the given gradients.

(a) $(1, 3), m = 2$

(b) $(-2, 5), m = -3$

(c) $(5, -1), m = \frac{1}{2}$

(d) $(6, 0), m = \frac{1}{3}$

3 Find the equation of each of the straight lines that pass through the following points.

(a) $(0, 5), (2, -1)$

(b) $(8, -4), (10, 4)$

(c) $(2, 5), (-3, 5)$

(d) $(2, 4), (0, -1)$

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Worksheet

Finding the equation of a straight line 3

Remember: $y = mx + c$ and $y - y_1 = m(x - x_1)$.

worked example

Find the equation of the straight line with gradient m = -2 that passes through the point (2, 6).

Steps	Solution
1. Determine x_1 and y_1 values and substitute into the formula.	$x_1 = 2$, $y_1 = 6$ $y - 6 = -2(x - 2)$
2. Simplify so that the equation is in the form $y = mx + c$.	$y - 6 = -2x + 4$ $y = -2x + 10$

exercise

1 Find the equation of each of the straight lines that pass through the following points and have the given gradients.

(a) (1, 5), $m = 3$

(b) (2, 3), $m = -2$

(c) (-1, 2), $m = -\frac{1}{2}$

(d) (3, -1), $m = \frac{3}{4}$

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